

Vacuum excited SPPs

Wade Naylor^{1, *}

¹*International College and Department of Physics,
Osaka University, Toyonaka, Osaka 560-0043, Japan*
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In a process analogous to photon-photon creation via the dynamical Casimir effect (DCE), after separating Maxwell's equations using Hertz vectors in a generalized Lorenz gauge, and which allows for both electric and magnetic time-dependent media, we discuss how surface plasmon polaritons (SPPs) can be created out of vacuum via the time-dependent variation of the medium. We find that vacuum excited SPPs dominate the photon creation rate for TM modes in a dielectric slab, modulated in time inside a cylindrical cavity. The possible experimental detection of vacuum SPPs is also discussed.

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Particle creation via the Schwinger-effect in static electric fields [1], in expanding universes [2] or from black hole evaporation [3] all have yet to be confirmed. However there is a related effect: the dynamical Casimir effect (DCE), first discussed by Moore [4] who showed that pairs of photons can be created in a cavity if one of the walls moves non-adiabatically. The number of photons created during a given number of parametric oscillations is proportional to $\sinh^2(2\omega t v/c)$, e.g., see [5], where v is the wall velocity and c is the speed of light. To overcome the fact that the mechanical properties of the material usually imply $v/c \ll 1$, there have been proposals other than mechanical oscillations. Calculations show that modulating a dielectric medium using a laser leads to particle creation by varying the optical path length of the cavity, e.g., see [6]. There are also related experiments underway in three-dimensions using centimeter-sized (microwave) cavities [7], where a laser is instead used to modulate the surface conductivity. Other methods use illuminated superconducting boundaries [8] and recently time varied inductance effects in one-dimensional quantum circuits have shown vacuum squeezing [9, 10]. Rotating analogs have also been investigated [11].

In this letter we discuss the possibility of the creation of vacuum excited surface plasmon polaritons (SPPs). In Fig. 1 the general idea is sketched, where a pulsed laser of an appropriate frequency can be used to vary the time dependence of the dielectric. The cavity is not *per se* required; however, a low temperature cavity will suppress thermal excitations and allows for comparison with calculations for photon-photon rates [6]. The fact that SPPs can be excited from vacuum fluctuations besides photons is much like the role SPPs play in the static Casimir force [12] with a metal/insulator/metal (MIM) heterostructure. We essentially generalize this to the dynamical case at first to the simpler case of a single metal/insulator (MI) interface and show that the time modulation of a dielectric leads not only to two-photon processes, but to vacuum excited SPPs that are comparable to the

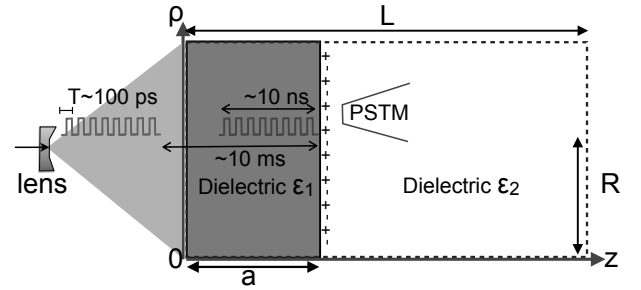


FIG. 1. A pulsed laser uniformly irradiates (via a lens) a dielectric slab (region 1) of thickness a and radius R in a superconducting cylindrical cavity (dotted lines) of length L . Vacuum excited SPPs (depicted by $- + - + - +$) can be detected by a photon scanning tunneling microscope placed in region 2. Alternatively a phase matched prism can be attached above the interface (in region 2) to detect surface plasmon resonances.

photon creation rate.

Maxwell's equations can be derived from the following Lagrangian:

$$\mathcal{L} = \frac{1}{2}\epsilon(t) \left(\frac{\partial}{\partial t} \Phi \right)^2 - \frac{1}{2} \frac{1}{\mu(t)} (\nabla \Phi)^2 - \frac{1}{2} m^2(t) \Phi^2 \quad (1)$$

($\epsilon_0 = \mu_0 = 1$). In the above we assume that the electric permittivity and magnetic permeability are time dependent, but constant in space: $\nabla \mu = \nabla \epsilon = 0$. Φ represents a TM field with generalized Neumann BCs in a cavity and the TE case (swapping $\epsilon \leftrightarrow \mu$) is represented by Ψ with Dirichlet BCs, e.g., see [13]. This Lagrangian is useful because the standard canonical Hamiltonian can be constructed [14], where the mass term, $m^2(t)$, represents the coupling of light to a time-dependent boundary (set to $m^2 = 0$ in what follows).

A convenient way to separate Maxwell's equations is using Hertz vectors; developed by Nisbet [15] for non-dispersive inhomogeneous media. However, here we generalize to the case of a constant isotropic, but time-dependent medium. It is then possible to show

that Maxwell's equations separate as

$$\begin{aligned}\varepsilon(t)\partial_t(\mu(t)\partial_t\mathbf{\Pi}_e) - \nabla^2\mathbf{\Pi}_e &= 0, \\ \mu(t)\partial_t(\varepsilon(t)\partial_t\mathbf{\Pi}_m) - \nabla^2\mathbf{\Pi}_m &= 0,\end{aligned}\quad (2)$$

where we use a generalized Lorenz gauge: $\mu(t)\partial_t(\varepsilon(t)A_0) + \nabla \cdot \mathbf{A} = 0$, cf. [15, 16]. In the above we have assumed both a zero permanent polarization and magnetization ($\mathbf{P}_0 = \mathbf{M}_0 = 0$) as well as zero bulk charges and currents ($\rho = 0, \mathbf{J} = 0$), although these can also be included in the Hertz method. Note the Lagrangian in Eq. (1) leads to the equations of motion for $\mathbf{\Pi}_e$ in Eq. (2) ($\mathbf{\Pi}_m$ is obtained by swapping $\mu \leftrightarrow \varepsilon$ in Eq. (1)). This approach generalizes other work [6, 16] which considered only time-dependent ε or μ . Further work for non-dispersive, inhomogeneous, conducting and time-dependent media: $\varepsilon(\mathbf{r}, t)$ and $\mu(\mathbf{r}, t)$, will be presented elsewhere.

Before discussing quantized SPPs we first need to find the classical solutions for a single interface (between two media) that lead to SPPs and are usually only discussed for TM modes [17]. With that goal in mind, we write the electric and magnetic fields in terms of the Hertz vectors:

$$\begin{aligned}\mathbf{E} &= \frac{1}{\varepsilon} \nabla \times (\nabla \times \mathbf{\Pi}_e) - \mu_0 \nabla \times \partial_t \mathbf{\Pi}_m, \\ \mathbf{B} &= \mu \nabla \times \frac{\partial \mathbf{\Pi}_e}{\partial t} + \mu_0 \nabla \times (\nabla \times \mathbf{\Pi}_m),\end{aligned}\quad (3)$$

allowing one to easily isolate TE and TM modes. For example, TM modes are defined by the parts $\mathbf{E}_{TM}, \mathbf{B}_{TM}$ coming from $\mathbf{\Pi}_e$ with $\hat{\mathbf{z}} \cdot \mathbf{B} = 0$, where a convenient choice of Hertz vectors are: $\mathbf{\Pi}_e = \Phi \hat{\mathbf{z}}, \mathbf{\Pi}_m = \Psi \hat{\mathbf{z}}$. For TM modes we obtain

$$\begin{aligned}\mathbf{E}_{TM} &= \frac{1}{\varepsilon} \partial_1 \partial_z \Phi \hat{\mathbf{e}}_1 + \frac{1}{\varepsilon} \partial_2 \partial_z \Phi \hat{\mathbf{e}}_2, \\ \mathbf{B}_{TM} &= \mu \partial_2 \partial_t \Phi \hat{\mathbf{e}}_1 - \mu \partial_1 \partial_t \Phi \hat{\mathbf{e}}_2\end{aligned}\quad (4)$$

with a similar expression for TE modes. These generalize the time-independent cases found, e.g., in [18].

In what follows take two half spaces in the $\hat{\mathbf{z}}$ -direction, where region 1 (the semiconductor slab) is to be metallic (M) during irradiation and region 2 an insulator (I) like air, creating an MI interface (later on we will discuss layered IMI and MIM heterostructures). In our proposed set up $\varepsilon_1(t)$ varies from \pm values and ε_2 remains constant (although for now it will be left more general). To make explicit the utility of the Hertz vector method we shall consider the radial propagation of SPPs in a cylindrical cavity with coordinates (ρ, θ, z) , sectional radius $\rho = R$ and length L , see Fig. 1.

To allow for time-dependent mode functions in either region we assume an ‘instantaneous’ set of mode functions [19]: $\Phi(\mathbf{r}, t) = \sum_{\mathbf{m}} Q_{\mathbf{m}}(t) \varphi_{\mathbf{m}}(\mathbf{r}; t)$ with $\mathbf{m} = (n, p, m)$, where t becomes a parameter: $\varphi(\mathbf{r}, t) \rightarrow \varphi(\mathbf{r}; t)$ and $\int_0^L dz \varepsilon(t) \varphi_{\mathbf{m}}(\mathbf{r}; t) \varphi_{\mathbf{n}}(\mathbf{r}; t) = (\varphi_{\mathbf{m}}, \varphi_{\mathbf{n}}) = \delta_{\mathbf{mn}}$ (orthonormal) and satisfy the wave equation: $\nabla^2 \varphi_{\mathbf{m}}(\mathbf{r}; t) + \varepsilon(t) \mu(t) \omega_{\mathbf{m}}^2(t) \varphi_{\mathbf{m}}(\mathbf{r}; t) = 0$. If we substitute this into the Lagrangian in Eq.

(1) using orthonormality of the mode functions and defining the conjugate momentum as $\Pi(\mathbf{r}, t) = \varepsilon(t) \sum_{\mathbf{m}} P_{\mathbf{m}}(t) \varphi_{\mathbf{m}}(\mathbf{r}; t)$ via a Legendre transform we then find an ‘effective’ Hamiltonian [20]: $H_{\text{eff}} = \sum_{\mathbf{m}} (P_{\mathbf{m}}^2 + \omega_{\mathbf{m}}^2(t) Q_{\mathbf{m}}^2) + \sum_{\mathbf{mn}} P_{\mathbf{m}} Q_{\mathbf{n}} \mathcal{M}_{\mathbf{mn}}(t)$, where $P_{\mathbf{m}} = \dot{Q}_{\mathbf{m}} - \mathcal{M}_{\mathbf{mn}} Q_{\mathbf{n}}$ and the intermode coupling matrix is $\mathcal{M}_{\mathbf{mn}} = \int_0^L dz \varepsilon(t) \varphi_{\mathbf{m}}(\mathbf{r}; t) \partial_t \varphi_{\mathbf{n}}(\mathbf{r}; t)$. However, for SPPs, using the definition in Eq. (5) below, it is easy to show that there are no intermode coupling terms: $\mathcal{M}_{\mathbf{mn}} = 0$, like for a uniform dielectric filling the entire cavity ($a = L$) [19]. Promoting these to operators, the ETCRs then become $[\hat{Q}_{\mathbf{m}}, \hat{P}_{\mathbf{n}}] = i \delta_{\mathbf{mn}}$, which are then equivalent to $[\hat{\Phi}_{\mathbf{m}}, \hat{\Pi}_{\mathbf{n}}] = i \delta_{\mathbf{mn}}$ when we quantize each degree of freedom (Φ, Ψ) .

To investigate SPPs for a single interface the ansatz:

$$\varphi_{\mathbf{m}}^{\text{SP}}(\mathbf{r}; t) = \begin{cases} A_1 e^{\kappa_1 m(z-a)} r_{\text{np}}(\mathbf{x}_{\perp}), & z < a; \varepsilon_1, \mu_1, \\ A_2 e^{-\kappa_2 m(z-a)} r_{\text{np}}(\mathbf{x}_{\perp}), & z > a; \varepsilon_2, \mu_2, \end{cases}\quad (5)$$

leads to the following dispersion relations:

$$\mathbf{k}_{\perp}^2 - \varepsilon_1 \mu_1 \frac{\omega_{\mathbf{m}}^2}{c^2} = \kappa_{1m}^2, \quad \mathbf{k}_{\perp}^2 - \varepsilon_2 \mu_2 \frac{\omega_{\mathbf{m}}^2}{c^2} = \kappa_{2m}^2 \quad (6)$$

where Eq. (2) was used in each region. For SPPs, the transverse modes are essentially unbounded and are simply given by $r_{\text{np}}(\mathbf{x}_{\perp}) = e^{i\mathbf{k}_{\perp} \cdot \mathbf{s}}$, where $\mathbf{s} = (\rho, \theta)$. In cylindrical coordinates the transverse Laplacian is defined by $\nabla_{\perp}^2 r_{\mathbf{k}_{\perp}} = -\mathbf{k}_{\perp}^2 r_{\mathbf{k}_{\perp}}$ with eigenvalue \mathbf{k}_{\perp}^2 . For the photon branch the modes are bounded where $r_{\text{np}}(\mathbf{x}_{\perp}) = \frac{1}{\sqrt{\pi}} \frac{1}{R J_{n+1}(x_{\text{np}})} J_n\left(x_{\text{np}} \frac{\rho}{R}\right) e^{in\theta}$ and x_{np} is the p th root of $J_n(x) = 0$ [21]. Note these mode functions are orthonormal: $(r_{\ell n}, r_{np}) = \delta_{\ell p}$.

Standard boundary conditions at an interface: $(\mathbf{D}_2 - \mathbf{D}_1) \cdot \hat{\mathbf{z}} = 0$ and $\hat{\mathbf{z}} \times (\mathbf{E}_2 - \mathbf{E}_1) = 0$ [21] then imply $A_1 = A_2$ and $\kappa_1(t)/\varepsilon_1(t) + \kappa_2(t)/\varepsilon_2(t) = 0$ which requires two dielectrics to be of opposite sign to generate SPPs [17]. Eliminating the z -dependent κ_i we find the following ‘electric’ dispersion relation:

$$k_{\perp} = |\mathbf{k}_{\perp}| = \frac{\omega_{\mathbf{m}}^{\text{SP}}}{c} \sqrt{\frac{\varepsilon_1 \varepsilon_2}{\varepsilon_1 + \varepsilon_2}} \times \left(\frac{\varepsilon_1 \mu_2 - \varepsilon_2 \mu_1}{\varepsilon_1 - \varepsilon_2} \right) \quad (7)$$

Note the symmetry in reversing regions 1 and 2. Then with $\mu_1 = \mu_2$ we get the standard result $(\omega_{\mathbf{m}}^{\text{SP}})^2 = \frac{k_{\perp}^2 c^2}{\varepsilon_2} (1 + \varepsilon_2/\varepsilon_1)$, where here we allow for time-dependent dielectrics.

It is also possible to show that magnetic SPPs exist for TE modes. Using the TE components of the Hertz vectors and using an ‘instantaneous’ equation like (5) for $\psi_{\mathbf{m}}(\mathbf{r}; t)$ along with $(\mathbf{B}_2 - \mathbf{B}_1) \cdot \hat{\mathbf{z}} = 0$ and $\hat{\mathbf{z}} \times (\mathbf{H}_2 - \mathbf{H}_1) = 0$ lead again to $A_1 = A_2$ but now with $\kappa_1(t)/\mu_1(t) + \kappa_2(t)/\mu_2(t) = 0$. Hence SPPs do exist for TE modes as long as for example, $\mu_1 < 0, \mu_2 > 0$, which can be achieved using split ring resonators, e.g., see [17]. Finally using $\mathbf{\Pi}_m$ in Eq. (2) leads to the ‘magnetic’ dispersion relation:

$$k_{\perp} = \frac{\omega_{\mathbf{m}}^{\text{SP}}}{c} \sqrt{\frac{\mu_1 \mu_2}{\mu_1 + \mu_2}} \times \left(\frac{\mu_1 \varepsilon_2 - \mu_2 \varepsilon_1}{\mu_1 - \mu_2} \right). \quad (8)$$

This result can be obtained from the ‘electric’ sector by swapping $\varepsilon_i \leftrightarrow \mu_i$ and when $\varepsilon_1 = \varepsilon_2$ the result simplifies to $(\omega_{\mathbf{m}}^{\text{SP}})^2 = \frac{k_{\perp}^2 c^2}{\mu_2} (1 + \mu_2/\mu_1)$.

To compare vacuum excited SPPs with some experimental proposals for photon creation using semiconductor slabs, e.g., see [5], we will consider TM modes. These have the following orthonormal mode functions in terms of the photon Hertz scalar:

$$\varphi_{\mathbf{m}}^{\text{ph}}(\mathbf{r}; t) = \begin{cases} A_1 \cos(k_{1m}z) r_{\text{np}}(\mathbf{x}_{\perp}), & 0 < z < a, \\ A_2 \cos(k_{2m}(L-z)) r_{\text{np}}(\mathbf{x}_{\perp}), & a < z < L. \end{cases} \quad (9)$$

where (using the same TM interface conditions as before) we find following transcendental equation

$$\frac{k_{1m} \tan(k_{1m}a)}{\varepsilon_1(t)} = \frac{k_{2m} \tan(k_{2m}[a-L])}{\varepsilon_2(t)} \quad (10)$$

for the eigenvalues. This agrees with the result in [6], but can be derived with the minimum of effort using Hertz vectors and generalized to arbitrary transverse section. Note that the photon dispersion relation (in this case for a cylindrical section) at any given time in regions $i = 1, 2$:

$$\omega_{\mathbf{im}}^{\text{ph}}(t) = \frac{c}{\varepsilon_i(t)} \sqrt{\left(\frac{x_{\text{np}}}{R}\right)^2 + k_{im}^2(t)} \quad (11)$$

must be equal at the interface implying that $\frac{1}{\varepsilon_1}(k_{1m}^2 + (\frac{x_{\text{np}}}{R})^2) = \frac{1}{\varepsilon_2}(k_{2m}^2 + (\frac{x_{\text{np}}}{R})^2)$. Both this constraint and the eigenvalue relation, Eq. (10), must be simultaneously satisfied [6].

The number of particles created can be obtained using the Heisenberg formalism, where the quantum field operator expansion is defined in terms of annihilation and creation operators as $\hat{\Phi}(\mathbf{r}, t) = \sum_{\mathbf{m}} [\hat{a}_{\mathbf{m}}^{\text{in}} u_{\mathbf{m}}^{\text{in}}(\mathbf{r}, t) + \hat{a}_{\mathbf{m}}^{\text{in}*} u_{\mathbf{m}}^{\text{in}*}(\mathbf{r}, t)]$. Before laser irradiation ($t < 0$) we have a stationary solution $u_{\mathbf{m}}^{\text{in}}(\mathbf{r}, t) = \frac{e^{-i\omega_{\mathbf{m}}^0 t}}{\sqrt{2\omega_{\mathbf{m}}^0}} \varphi_{\mathbf{m}}(\mathbf{r}; t_0)$, while for $0 \leq t \leq t_1$ (during irradiation) we again expand in an ‘instantaneous’ basis: $u_{\mathbf{m}}^{\text{in}}(\mathbf{r}, t) = \sum_{\mathbf{m}} Q_{\mathbf{m}}(t) \varphi_{\mathbf{m}}(\mathbf{r}; t)$ with initial conditions $Q_{\mathbf{m}}(0) = \frac{1}{\sqrt{2\omega_{\mathbf{m}}}}$ and $\dot{Q}_{\mathbf{m}}(0) = -i\sqrt{\frac{\omega_{\mathbf{m}}}{2}}$ derived from continuity of $u_{\mathbf{m}}$ and $\dot{u}_{\mathbf{m}}$ at $t = 0$. When this is substituted into Eq. (2) and using orthonormality of $\varphi_{\mathbf{m}}(\mathbf{r}; t)$ we find an equation for an uncoupled set of time-dependent harmonic oscillators: $\ddot{Q}_{\mathbf{m}} + \omega_{\mathbf{m}}^2(t) Q_{\mathbf{m}} = 0$, for $\mathcal{M}_{\mathbf{mn}} = 0$.

For times $t > t_1$ we expand the mode functions in terms of a new stationary eigenfrequency $\omega_{\mathbf{m}}^1$: $u_{\mathbf{m}}^{\text{out}}(\mathbf{r}, t) = \frac{e^{-i\omega_{\mathbf{m}}^1(t-t_1)}}{\sqrt{2\omega_{\mathbf{m}}^1}} \varphi_{\mathbf{m}}(\mathbf{r}; t_1)$, where the two sets of mode functions are related by a Bogolyubov transformation: $u_{\mathbf{m}}^{\text{out}}(\mathbf{r}, t) = \sum_{\mathbf{n}} [\alpha_{\mathbf{nm}} u_{\mathbf{n}}^{\text{in}}(\mathbf{r}, t) + \beta_{\mathbf{nm}}^* u_{\mathbf{n}}^{\text{in}*}(\mathbf{r}, t)]$, where $\alpha_{\mathbf{nm}} = ((u_{\mathbf{m}}^{\text{out}}, u_{\mathbf{n}}^{\text{in}}))$ and $\beta_{\mathbf{nm}} = -((u_{\mathbf{m}}^{\text{out}}, [u_{\mathbf{n}}^{\text{in}}]^*))$ and the scalar invariant product is defined $((\phi, \psi)) = -i \int_{\text{cavity}} d^3x (\phi \dot{\psi}^* - \dot{\phi} \psi^*)$. For $\mathcal{M}_{\mathbf{mn}} = 0$ and assuming the background field (the laser) leads to shifts in frequency near to parametric resonance then $\omega_{\mathbf{m}}^2(t) \sim \omega_{0\mathbf{m}}^2 + \Delta\omega_{0\mathbf{m}}^2 = \omega_{0\mathbf{m}}^2(1 + \kappa \sin(2\omega_{0\mathbf{m}}t))$, where $\omega_{0\mathbf{m}}$ is the driving frequency. In the late time limit we then have enhancement $N_{\mathbf{m}} = |\beta_{\mathbf{mm}}(t)|^2 = \sinh^2(\omega_{0\mathbf{m}}\kappa t/2)$ [22].

For simplicity consider $\mu_1 = \mu_2$, then the time-dependent ‘electric’ SPP dispersion relation, see below

Eq. (7), for ε_2 a constant and $\varepsilon_{\min} < \varepsilon_1(t) < \varepsilon_{\max}$ with $\varepsilon_{\min} < 0$ gives $1 + \frac{\varepsilon_2}{\varepsilon_1(t)} \sim 1 + \chi - \kappa \sin(\omega_{0\mathbf{m}}t)$, where χ is an overall time-independent frequency shift. $\chi = -1$ then generates particles with enhancement factor

$$N_{\mathbf{m}}^{\text{sp}} = \sinh^2\left(\frac{k_{\perp}^2 c^2}{\omega_{0\mathbf{m}} 2\varepsilon_2} t\right), \quad (12)$$

which is one of the main results of this letter. We can now compare this to the ω_{011} TM mode (the lowest frequency cylindrical mode [13]) where in the limit of $a \ll L$ the photon eigenvalues shift by $\Delta\omega_{\mathbf{m}}^2(t) = \frac{2x_{\text{np}}^2 c^2}{R^2 \varepsilon_2} \frac{a}{L} \left[\frac{\varepsilon_2}{\varepsilon_1(t)} - 1\right] + \mathcal{O}\left[\frac{a^2}{L^2}\right]$. Assuming the coupling matrix $\mathcal{M}_{\mathbf{mn}} = 0$ which can be achieved for certain cavity geometries [6], then the parametric enhancement for the photon branch is achieved by choosing $\frac{\varepsilon_2}{\varepsilon_1(t)} - 1 \sim \chi - 1 + \kappa \sin(\omega_{0\mathbf{m}}t)$ and leads to (for $\chi = 1$): $N_{\mathbf{m}}^{\text{ph}} = \sinh^2\left(\frac{x_{\text{np}}^2 c^2}{R^2 \omega_{0\mathbf{m}} \varepsilon_2 L} t\right)$. For a cylindrical cavity the SPP creation rate dominates the photon rate if $k_{\perp}^2 \gg (x_{\text{np}}^2/R^2)(a/L)$, which for example with a radius $R = 2.5$ cm and length $L = 10$ cm, then if $a/L \sim \mathcal{O}[10^{-4}]$ and $x_{01} = 2.4048$ implies $k_{\perp}^2 \gg 1/25$ or $k_{\perp} \gg 1/5$ which is easily achieved for SPPs of micro to nanometer wavelength, cf. $\lambda_{\text{sp}} = 2\pi/\text{Re}[k_{\perp}]$ (we discuss the propagation length $L = (2\text{Im}[k_{\perp}])^{-1}$ later).

To realistically modulate the permittivity, $\varepsilon_1(t)$, we could use an appropriately doped semiconductor (with two well defined energy levels within the band gap) via Rabi oscillations, e.g, see [23]. However, in this case, $\varepsilon_{\min} > 0$ (implying frequency shifts $\chi > 0$) and there are no SPP solutions. For laser pulses with an energy ($h\lambda/c$) above the band gap, a time varying bulk conductivity, $\rho(t)$, would be generated leading to a modulated permittivity with $\varepsilon_{\min} < 0$ (shifts $\chi < 0$). Such a conductivity can be included in Maxwell’s equations, Eq. (2), using $\mathbf{J} = \sigma(t)\mathbf{E}$ and hence this only affects TM modes (surface charges, $\sigma(t)$ are present for both TE and TM modes [13]). An alternative using only positive definite modulations for $\varepsilon_1(t)$ using IMI heterostructures, is discussed later.

Possible ways to detect vacuum excited SPPs would be to use near field microscopy with a photon scanning tunneling microscope, e.g., see [17], where the microscope is placed on the opposite vacuum/air side of the dielectric slab, see Fig. 1. Usually, to generate SPPs a monochromatic light source is sent into a prism placed above an interface (region 2) with total internal reflection at angle θ_i , where their presence is then verified by finding a decrease in emitted power at θ_i . However, the time reversal of this process is equivalent to the creation of vacuum excited SPPs and hence might lead to the intriguing result that a signal can be detected from an empty prism during the time modulation of ε_1 . We should of course require that the pulsed laser itself does not generate SPPs, as can be arranged by uniformly irradiating the dielectric slab at a right angle of incidence, see Fig. 1.

It is also possible to consider more complicated multilayer regions such as insulator/metal/insulator

(IMI) heterostructures or MIMs (made of three regions 1, 2, and 3, respectively). It is easy to verify (when $\mu_1 = \mu_2$) that our results follow through as in the standard case, but now ε_1 , for example, is time dependent: $e^{-4\kappa_2 a} = \frac{\kappa_2/\varepsilon_2 + \kappa_1/\varepsilon_1(t)}{\kappa_2/\varepsilon_2 - \kappa_1/\varepsilon_1(t)} \times \frac{\kappa_2/\varepsilon_2 + \kappa_3/\varepsilon_3}{\kappa_2/\varepsilon_2 - \kappa_3/\varepsilon_3}$. In particular, IMIs develop long range SPPs for ‘even’ mode solutions [17] when the metal thickness $a \rightarrow 0$ and may allow for easier detection of vacuum excited SPPs. This also shows that SPP solutions are possible with a modulation of ε_1 positive definite, because region 2 (metal) has $\varepsilon_2 < 0$, as we mentioned earlier.

We also discussed how SPPs can occur for TE modes if we assume: $\mu_i < 0$ (a metamaterial with $\varepsilon_i, \mu_i < 0$ would simultaneously allow for both TE and TM mode SPPs). Our results for parametric enhancement go through the same: $N_{\mathbf{m}}^{\text{sp}} = \sinh^2 \left(\frac{k_{\perp}^2 c^2}{\omega_{0\mathbf{m}} 2\mu_2} t \right)$, with μ_2 replacing ε_2 . We expect this rate to also be comparable to the TE photon-photon rate if μ_1 were varied somehow in time at GHz frequencies. It might be of further interest to try to design experiments in centimeter sized cavities using split ring resonators and wire rods that are then modulated in time. This may allow for easier detection by precisely controlling the SPP wavelength, λ_{sp} .

In summary, in this letter we discussed how SPPs might be excited from vacuum for the case of a dielectric slab which behaves as an insulator/insulator (II) interface and changes to an MI interface during irradiation. We separated Maxwell’s equations using the Hertz vectors approach for time-dependent media finding both electric and magnetic SPP solutions. For parametric oscillations of the dielectric inside a superconducting microwave cylindrical cavity (cooled to suppress thermal excitations) our calculations show that vacuum excited SPPs can dominate over the photon-photon rate. The results for the photon creation rate in a cylindrical cavity generalizes those found for the rectangular case [6].

We used an ‘instantaneous’ basis, which treats time as a parameter and might warrant some concern regarding quantization. Firstly, we are not considering moving boundaries which lead to non-separable Hilbert spaces; however, these can be remedied by a coordinate transformation to a stationary problem [24]. Secondly, we can also expand the mode functions using an ‘initial mode’ basis which lead to near identical results when compared to the ‘instantaneous’ method (these were found to be identical when there are no intermode couplings, $\mathcal{M}_{\mathbf{mn}} = 0$) [14]. We should also include dissipative effects, because $\text{Im}[\varepsilon_1] \neq 0$ for both SPPs and photons. However, for SPPs this diminishes the propagation length via $L = (2\text{Im}[k_{\perp}])^{-1}$, not the production rate. There are other issues relating to de-tuning of the resonant frequencies arising from dissipation, but these are out of the scope of the current letter and will be addressed elsewhere.

Finally, given that there are no intermode couplings ($\mathcal{M}_{\mathbf{mn}}=0$) these SPPs behave as ‘pure’ modes

and, for example, have well defined squeezing parameters, which could have applications in quantum information. It would also be interesting to investigate the vacuum excitation of volume plasmon polaritons (VPPs). Due to their longitudinal nature they are not excited by light and hence require particle impact at the interface, e.g., see [17]. However, vacuum excitations might be achieved dynamically by firing clusters of ‘neutral’ Argon atoms at a sample of material [25], or by placing the sample on a high frequency piezo, e.g., see [26]. We hope to comment on this and related issues in future work.

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* Email: naylor@phys.sci.osaka-u.ac.jp

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